HW 19 Fluids in Motion

1) A cylindrical tank containing water of density $1000kg/m^3$ is filled to a height of 0.70m and placed on a stand as shown in the cross section above. A hole of radius 0.001m in the bottom of the tank is opened. Water then flows through the hole and through an opening in the stand and is collected in a tray 0.30m below the hole. At the same time, water is added to the tank at an appropriate rate so that the water level in the tank remains constant.

a) Calculate the speed at which the water flows out of the hole.

$$P + \frac{1}{2}\rho v^2 + \rho g \triangle y = P + \frac{1}{2}\rho v^2 + \rho g \triangle y$$

$$\frac{1}{2}\rho v^2 = \rho g \triangle y$$

$$\frac{1}{2}(1000kg/m^3)v^2 = (1000kg/m^3)(9.8m/s^2)(0.7m)$$

$$v^2 = 2(9.8m/s^2)(0.7m)$$

$$v = \sqrt{2(9.8m/s^2)(0.7m)}$$

$$v = \boxed{3.70m/s}$$

b) Calculate the volume rate at which water flows out from the hole.

$$Area = \pi r^2$$

$$Area = \pi(0.001m)^2$$

$$Rate_{flow} = Area \times Velocity$$

$$Rate_{flow} = (3.14 \times 10^{-6}m^2)(3.17m/s)$$

$$Rate_{flow} = \boxed{1.16 \times 10^{-5}m^3/s}$$

c) Calculate the volume of water collected in the tray in t=2.0 minutes.

$$Volume = Rate_{flow} \times Time$$

$$Volume = (1.16 \times 10^{-5}m^3/s)(120sec)$$

$Volume = \boxed{0.00139 m^3}$

d) Calculate the time it takes for a given droplet of water to fall 0.25m from the hole.

$$\triangle y = v_o t + \frac{1}{2} g t^2$$

$$-0.25m = (-3.7m/s)t + \frac{1}{2}(-9.8m/s)t^2$$

$$(3.7m/s)t + \frac{1}{2}(9.8m/s)t^2 - 0.25m = 0$$

Using the quadratic formula, we find that $t = \boxed{0.0624s}$

2) A drinking fountain projects water at an initial angle of 50^o above the horizontal, and the water reaches a maximum height of 0.150m above the point of exit. Assume air resistance is negligible.

a) Calculate the speed at which the water leaves the fountain.

$$\triangle x = v_o t + \frac{1}{2} a t^2$$

$$-0.15m = \frac{1}{2}(-9.8m/s^2)(t^2)$$

$$t = 0.175 sec$$

$$\triangle x = v_o t + \frac{1}{2} a t^2$$

$$0.15m = v_o(0.175sec) + \frac{1}{2}(-9.8m/s^2)(0.175sec)^2$$

$$v_o = 1.714m/s$$

$$v \sin 50^o = 1.714m/s$$

$$v = \boxed{2.24m/s}$$

b) The radius of the fountain's exit hole is 4.00×10^{-3}m. Calculate the volume rate of flow of the water.

$$Rate_{flow} = Area \times Velocity$$

$$Rate_{flow} = \pi(0.004m)^2$$

$$Rate_{flow} = \boxed{0.0001126m^3/s \; or \; 1.126 \times 10^{-4}m^3/sec}$$

c) The fountain is fed by a pipe that at one point has a radius of 7.00×10^{-3}m and is 3.00 m below the fountain's opening. The density of water is $1.0 \times 10^3 kg/m^3$. Calculate the gauge pressure in the feeder pipe at this point.

$$A_1V_1 = A_2V_2$$

$$\pi(0.004m)^2(2.24m/s) = \pi(0.007m/s)^2(v)$$

$$v = 0.7314m/s$$

$$P + \frac{1}{2}\rho v^2 + \rho g \triangle y = P + \frac{1}{2}\rho v^2 + \rho g \triangle y$$

$$\frac{1}{2}(1000kg/m^3)(2.24m/s)^2 + (1000kg/m^3)(9.8m/s^2)(3m) = P_{gauge} + \frac{1}{2}(1000kg/m^3)(0.7314m/s)^2$$

$$P = \boxed{31,641.3Pa}$$